Discrete-Time Variable Structure Controller with a Decoupled Disturbance Compensator and Its Application to a CNC Servomechanism

Yongsoon Eun, Jung-Ho Kim, Kwangsoo Kim, and Dong-II Cho, Member, IEEE

Abstract—This paper develops a new approach in regulation and tracking, combining a disturbance compensator with a controller, both of which are formulated in the discrete-time domain using the variable structure concept. For single-input single-output, linear time-invariant systems, an algorithm for exactly decoupling the disturbance estimation dynamics from the sliding mode dynamics is developed. This allows the two dynamic modes to be tuned separately. It is shown that the developed approach preserves the robustness properties of the sliding mode and asymptotically achieves zero tracking error, in the presence of external disturbances and parametric uncertainties. Both computer simulation results and experimental results using a CNC milling machine are used to demonstrate those properties of the proposed method in both regulation and tracking.

Index Terms—Decoupled disturbance compensator (DDC), discrete-time variable structure control (DVSC), separation principle.

I. INTRODUCTION

VARIABLE structure control (VSC) was first presented by Utkin [1]. The controller is a function of more than two structures and gives some desirable closed-loop properties. The desirable features include invariance, order reduction, and robustness against parameter variations and disturbances.

The stability and robustness of continuous-time VSC is guaranteed using the concept of switching function $s(x(t))$ and Lyapunov stability theory. Let the Lyapunov function $V(t)$ be $\frac{1}{2}s(x(t))^2$ with the switching function $s(x(t))$. For the stability and robustness of the closed-loop system, the control is chosen to satisfy the well-known sliding condition

$$\frac{dV(t)}{dt} = s(x(t)) \frac{ds(x(t))}{dt} < 0,$$ (1)

In the discrete-time domain, (1) can be interpreted as

$$\sum_{k} (s[k + 1] - s[k])s[k] < 0.$$ (2)

References [2] and [3] showed that (2) is necessary but not sufficient for achieving a sliding motion of discrete variable structure systems, and proposed a more strict condition as follows:

$$|s[k + 1]| < |s[k]|.$$ (3)

Reference [4] formulated the concept of discrete sliding mode, and used the reaching law approach proposed in [5] to satisfy the reaching condition for stability and convergence of $s(k)$. However, the invariance property is not achieved, and the system suffers from the chatter of switching function when an external disturbance exists. Reference [6] suggests a periodic convergence law (PCL) to solve this inherent steady state chatter problem, but the PCL strategy does not consider robustness. For robustness, [7] uses a predictive corrective scheme for an input–output plant model. Another approach is to use a disturbance compensator, but this adaptive sliding control method may have an adverse effect when an initial tracking error exists. Reference [8] solves this problem by using $s(k)$ and $s[k-1]$ for the adaptation law instead of using $s(k)$, but [8] restricts the method to continuous canonical form and uses a pseudodiscretized model, which in turn results in an unwanted chatter.

This paper designs a decoupled disturbance compensator (DDC) extending the results of [8], and formulates a methodology for combining with the discrete sliding mode concept to satisfy the sliding condition, thus also extending the results of [4].

The separation between a controller and other embedded algorithms (such as a disturbance compensator or state observer) is a desirable property. Reference [9] derives a decoupling method between feedback linearization and state observer dynamics. In this paper a decoupling method between a discrete-time variable structure controller and a disturbance compensator is developed. An algorithm is developed to decouple exactly, the disturbance estimation dynamics from the sliding mode dynamics, allowing tuning the two dynamic modes separately. For example, the disturbance estimation dynamics can be tuned fast within sampling constraints and with no overshoot, and the sliding mode dynamics can be tailored to achieve certain performance and robustness requirements. It is shown, both in regulation and tracking problems, that the proposed method achieves the sliding condition in the presence of external disturbances and parametric uncertainties. The conventionally assumed upper bound restriction on the unknowns is also relaxed to the restriction of the
changing rate of the unknowns. It is also shown that tracking error goes to zero asymptotically when the unknowns are constant.

The outline of this paper is as follows. In Section II, a brief description of the conventional discrete-time variable structure control is given. In Section III, a disturbance estimation law and a method for exactly decoupling the disturbance estimation dynamics from the sliding mode dynamics are developed. In Section IV, the proposed method is demonstrated using a CNC milling machine.

II. DISCRETE-TIME VARIABLE STRUCTURE CONTROLLER

Consider the SISO LTI system described in (4)

\[
\mathbf{x}(k+1) = (A + \Delta A)\mathbf{x}(k) + b\mathbf{u}(k) + f(k)
\]

where \( \mathbf{x} \in \mathbb{R}^n \) is the state vector, \( b \) and \( f \) are \( n \times 1 \) column vectors, \( A \) and \( \Delta A \) are \( n \times n \) square matrices, and \( u \) is a scalar.

**Definition 1—Matching Condition:** The LTI system (4) satisfies the matching condition if there exists an \( n \times 1 \) row vector \( \bar{A} \) and a scalar \( \bar{f} \) such that \( \Delta A = b\bar{A} \), and \( f = b\bar{f} \).

The following two assumptions hold throughout this paper.

**Assumption 1:** The \((A, b)\) pair is completely controllable in (4).

**Assumption 2:** The matching condition holds for the system (4).

With Assumption 2, (4) can be rewritten in the discrete-time form

\[
\mathbf{z}(k+1) = \mathbf{A}_z\mathbf{z}(k) + b[u(k) + d(k)].
\]

In (5), the disturbance \( d(k) \) is constructed as \( d(k) = b\bar{A}\mathbf{z}(k) + \bar{f}(k) \), including the parameter uncertainty \( \Delta A \), and the external disturbance vector \( f(k) \). The switching function \( s(k) \) is defined as follows:

\[
s(k) = c^T\mathbf{z}(k) = 0.
\]

The control objective of the discrete-time variable structure control (DVSC) is to achieve \( s(k+1) = s(k) = 0 \). When \( s(k) = 0 \), i.e., when the state vector remains on the sliding surface, the closed-loop dynamics become:

\[
\mathbf{z}(k+1) = \left[ I - b(c^Tb)^{-1}c^T \right]A\mathbf{z}(k).
\]

The state vector \( \mathbf{z}(k) \) remains on the sliding surface defined in (6) and converges to the origin in the state space, if \( [I - b(c^Tb)^{-1}c^T]A \) is a contractive matrix. The conventional DVSC given in [4] is as follows.

**Theorem 1—[4]:** For an LTI system given by (5), choosing the control law as

\[
u(k) = (c^Tb)^{-1} \left[ -c^TA\mathbf{z}(k) + (1-qT)s(k) + (\eta + \varepsilon)T \text{sgn}(s(k)) \right]
\]

(8) with \( q, \eta, \varepsilon, 1-qT > 0 \), the following closed-loop sliding mode dynamics are satisfied:

\[
s(k+1) = (1-qT)s(k) - (\eta + \varepsilon)T \text{sgn}(s(k)) + c^Tbd(k),
\]

(9)

The authors show in [4] that the conventional DVSC method can bound \( s(k) \) in a certain region in the state space when the disturbance satisfies \( [c^Tbd(k)] < \eta T \). However, the control (8) does not achieve asymptotic convergence to \( s(k) = 0 \). Furthermore, the invariant property of continuous time VSC is not retained. The control (8) does not make \( s(k) \) converge to zero, and results in sliding surface or switching function chatter. The chatter magnitude is approximately \( \pm (\eta T + \varepsilon T)/(2-qT) \). The control (8) also results in the deviation of \( s(k) \) from zero under a constant external disturbance.

III. DISCRETE-TIME VARIABLE STRUCTURE CONTROLLER WITH DECOUPLED DISTURBANCE COMPENSATOR

A new method is proposed to improve the above-discussed problems of the conventional DVSC [4]. This proposed method is applicable to both regulation and tracking problems. In this paper, the tracking problem is considered first, and by setting the reference vector in the tracking problem to zero, the properties of the regulation problem are derived. The tracking error vector is defined with a reference vector as follows:

\[
\text{reference vector: } \mathbf{z}_r(k) = [x_{r1}(k) \cdots x_{rn}(k)]^T
\]

(10)

**tracking error vector:**

\[
\mathbf{z}(k) = \mathbf{z}(k) - \mathbf{z}_r(k).
\]

(11)

The switching function is defined with the error vector and an \( n \times 1 \) column vector \( c \) as follows:

\[
\text{switching function: } s(k) = c^T\mathbf{z}(k).
\]

(12)

To analyze sliding mode dynamics when \( s(k) = 0 \), using (5) and (12)

\[
s(k+1) = c^TA\mathbf{z}(k) + bn(k) + bd(k) - \mathbf{z}_r(k+1)
\]

(13)

A linear feedback control \( u(k) \) is obtained from (13)

\[
u(k) = \mathbf{z}(k) - \frac{1}{c^Tb}c^T(A\mathbf{z}(k) - \mathbf{z}_r(k+1)).
\]

(14)

Combining (14) and (5) with (11)

\[
\mathbf{z}(k+1) = \left[ I - b(c^Tb)^{-1}c^T \right]A\mathbf{z}(k) - \left[ I - b(c^Tb)^{-1}c^T \right]A\mathbf{z}_r(k+1).
\]

(15)

**Theorem 2—Asymptotic Stability:** For system (5), the sliding surface (12) or the vector \( c \) can be designed to give the asymptotic convergence of tracking error vector to zero, with Assumption 1. The tracking error dynamics are governed by

\[
\mathbf{z}(k+1) = \left[ I - b(c^Tb)^{-1}c^T \right]A\mathbf{z}(k).
\]

(16)
Proof: By Assumption 1, let system (5) be in controllable canonical form in which

\[
A = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
a_1 & a_2 & a_3 & \cdots & a_n
\end{bmatrix}, \quad b = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
1
\end{bmatrix}
\]

(17)

e^T = [c_1 \ c_2 \ \cdots \ c_{n-1}].

Each component of the state reference vector \( x_r(k) \) is related by \( A \) as follows:

\[
x_r(k+1) = x_r(i+1)(k), \quad 1 \leq i \leq n-1.
\]

(18)

Using (18), the last part in (15) can be rewritten as

\[
\xi_r(k+1) = A\xi_r(k) = \begin{bmatrix}
0 \\
\vdots \\
0
\end{bmatrix} x_{r:n}(k+1) - \sum_{i=1}^{n} a_i x_{ri}(k)
\]

(19)

With (15) and (19), (16) is derived by a direct substitution. Indeed, the last term of (15) disappears by the following:

\[
\left[ I - b(c^T b)^{-1} c^T \right] A\xi_r(k) = \begin{bmatrix}
0 \\
\vdots \\
0
\end{bmatrix} x_{r:n}(k+1) - \sum_{i=1}^{n} a_i x_{ri}(k)
\]

(20)

To achieve the asymptotic stability of (16), the matrix \( \left[ I - b(c^T b)^{-1} c^T \right] A \) is required to be contractive. By (17), this matrix reduces further to \( \left[ I - b^2 c^T \right] A \), and with Assumption 1, the eigenvalues can be assigned arbitrarily by a suitable choice of vector \( c \). ■

Corollary 1: For system (5), sliding mode dynamics in regulation problem can be designed to give the asymptotic convergence of state vector to zero by setting the state reference vector to zero. The sliding dynamics are given by (7) or (16).

Theorem 2 summarizes the tracking error behavior once the closed-loop system is in the sliding mode. To achieve the sliding mode, a new control law with a disturbance estimation law is proposed in Theorem 3.

\[
\xi_r = \begin{bmatrix}
\xi_r \\
\xi_r
\end{bmatrix}, \quad \xi_r(k+1) = A\xi_r(k) = \begin{bmatrix}
0 \\
\vdots \\
0
\end{bmatrix} x_{r:n}(k+1) - \sum_{i=1}^{n} a_i x_{ri}(k)
\]

(19)

with \( \hat{d}(k) = \hat{d}(k) - \hat{d}(k) \), where \( \hat{d}(k) \) is the estimated disturbance, \( \hat{d}(k) \) is the disturbance estimation error, and \( \eta, q, g \) are constants, the following closed-loop sliding mode dynamics and disturbance estimation error dynamics are satisfied:

\[
s(k+1) = qs(k - \eta \text{sgn}(\xi_r(k))) + c^T b \hat{d}(k)
\]

(23)

\[
\hat{d}(k+1) = (1 - g)\hat{d}(k) + \hat{d}(k+1) - \hat{d}(k).
\]

(24)

In (23) and (24), \( s(k) \) dynamics and \( \hat{d}(k) \) dynamics are decoupled.

Proof: The proof is rather straightforward, using (11) and substituting (21) into (5), (23) is verified as follows:

\[
s(k+1) = c^T g x(k+1) - c^T \xi_r(k+1) = c^T A \xi_r(k) + c^T b u(k) + c^T b d(k) - c^T \xi_r(k+1) = q \xi_r(k) - \eta \xi_r(k) + c^T b \hat{d}(k).
\]

(23)

By substituting (22) and (23) into the equation \( \hat{d}(k) = \hat{d}(k) - \hat{d}(k) \), (24) is derived as follows:

\[
\hat{d}(k+1) = \hat{d}(k+1) - \hat{d}(k+1) = \hat{d}(k+1) - \hat{d}(k) - \{ c^T b \}^{-1} [g q \xi_r(k+1) - q \xi_r(k) - \eta \text{sgn}(\xi_r(k))]
\]

\[
= \hat{d}(k+1) - \hat{d}(k) + \hat{d}(k) - \{ c^T b \}^{-1} [g q \xi_r(k+1) - q \xi_r(k) - \eta \text{sgn}(\xi_r(k))]
\]

\[
= \hat{d}(k+1) - \hat{d}(k) + \{ c^T b \}^{-1} [g q \xi_r(k+1) - q \xi_r(k) - \eta \text{sgn}(\xi_r(k))]
\]

\[
= \hat{d}(k+1) - \hat{d}(k) + \{ c^T b \}^{-1} [g q \xi_r(k+1) - q \xi_r(k) - \eta \text{sgn}(\xi_r(k))]
\]

(24)

The discrete variable structure control (DVSC) of (23) and the decoupled disturbance compensator (DDC) of (24) do not assume the upper bound of disturbance as in the conventional VSC method. Instead it is assumed that the changing rate of disturbance is bounded. Under this assumption, the quantitative property of \( \hat{d}(k) \) is examined in Lemma 1.

Lemma 1: For (24) in Theorem 3, if \( |d(k+1) - d(k)| < m \) holds for all \( k \) with \( |1 - g| < 1 \) and for some positive constant \( m \), then there exists some \( k_0 \) such that \( |\hat{d}(k)| \leq m g / 2 \) for all \( k > k_0 \) regardless of \( \hat{d}(0) \).

Proof: Divide \( \hat{d}(k) \) into \( \hat{d}_1(k) + \hat{d}_2(k) \) with \( \hat{d}_1(0) = 0 \) and \( \hat{d}_2(0) = \hat{d}(0) \). Then, (24) can be interpreted as

\[
\hat{d}_1(k+1) = (1 - g)\hat{d}_1(k) + d(k+1) - d(k)
\]

(25)

\[
\hat{d}_2(k+1) = (1 - \hat{g})\hat{d}_2(k).
\]

(26)

If \( |\hat{d}_1(k)| < m g / 2 \), then, from (25) and the assumption \( |d(k+1) - d(k)| < m \) the following inequality holds:

\[
\hat{d}_1(k+1) < \hat{m} g / 2 - m < \hat{d}_1(k+1)
\]

\[
(1 - g) \hat{m} g / 2 + m < \hat{d}_1(k+1)
\]

(27)

which means \( |\hat{d}_1(k+1)| < m g / 2 \). This holds for all \( k > 0 \) since \( \hat{d}_1(0) \) is zero. It is clear from (26) and the assumption \( |1 - g| < 1 \) that \( \hat{d}_2(k) \) can be arbitrarily small for some \( k_0 \). ■
Fig. 1. State variables $x_1$ and $x_2$.

It is shown in Lemma 1 that the estimation error $\hat{d}(k)$ in (24) asymptotically converges to zero if the disturbance is constant or slowly varying. By Lemma 1, in (23), $s(k)$ goes to zero asymptotically if the disturbance is constant. The switching function $s(k)$ is also bounded by the proposed control, which is given in Theorem 4.

**Theorem 4—Robust Stability to Disturbance:** For the system (5), if the following a)–c) hold, then the closed-loop system described by (23) and (24) is stable:

1) $0 \leq q \leq 1$, $0 < g < 1$;
2) $|\hat{d}(k+1) - \hat{d}(k)| < m$ holds for all $k$ for some constant $m > 0$;
3) $c^Tb(m/g) < \eta$.

**Proof:** From (24) and the assumptions 1) and 2), $|\hat{d}(k)| \leq m/g$. Define $\nu(k)$ such that $\nu(k) = c^Tb(k)$.

Then each $\nu(k)$ satisfies $|\nu(k)| \leq c^Tb(m/g) < \eta$ by the condition 3). The switching function dynamics are $s(k+1) = qs(k) - \eta \text{sgn}(s(k)) + \nu(k)$. If $s(k) \geq c^Tb(m/g) + \eta > 0$, then the following results are obtained from the switching function dynamics with the condition 1), and $q - 1 \leq 0$:

$$s(k+1) - s(k) = (q - 1)s(k) - \eta + \nu(k)$$

$$\leq (q - 1)\left(c^Tb\frac{m}{g} + \eta\right) - \eta + \nu(k) < 0$$

$$s(k+1) + s(k) = (q + 1)s(k) - \eta + \nu(k)$$

$$\geq q\left(c^Tb\frac{m}{g} + \eta\right) + c^Tb\frac{m}{g} + \nu(k) > 0.$$ 

These are combined to obtain $s(k+1)^2 < s(k)^2$. When $s(k) < -c^Tb(m/g) - \eta < 0$, with the similar procedure, $s(k+1)^2 < s(k)^2$ is obtained. If $0 < s(k) < c^Tb(m/g) + \eta$, then, from the conditions 1) and 3), i.e., the conditions $0 < q < 1$ and $|\nu(k)| \leq c^Tb(m/g) < \eta$, the following inequality holds:

$$-c^Tb\frac{m}{g} - \eta < \nu(k) - \eta < s(k+1)$$

$$< q\left(c^Tb\frac{m}{g} + \eta\right) + \nu(k) - \eta < c^Tb\frac{m}{g} + \eta.$$
On the other hand, if \(-c^T \mathbf{u}(m/g) - \eta < \mathbf{s}(k) < 0\), then similarly, the inequality
\[
-c^T \mathbf{b} \frac{m}{g} - \eta < -q \left( c^T \mathbf{b} \frac{m}{g} + \eta \right) + \mathbf{v}(k) 
\]
\[
< \mathbf{s}(k+1) < \eta + \mathbf{v}(k) < c^T \mathbf{b} \frac{m}{g} + \eta
\]
is obtained. These results are summarized below
\[
\mathbf{s}(k+1)^2 < \mathbf{s}(k)^2, \quad \text{if } |\mathbf{s}(k)| \geq c^T \mathbf{b} \frac{m}{g} + \eta
\]
\[
|\mathbf{s}(k+1)| < c^T \mathbf{b} \frac{m}{g} + \eta, \quad \text{if } |\mathbf{s}(k)| < c^T \mathbf{b} \frac{m}{g} + \eta.
\]

Theorem 4 states that each switching function remains smaller than \(c^T \mathbf{u}(m/g) + \eta\) regardless of the size of the disturbance, if the conditions 1)–3) hold. When the disturbance varies slowly, \(m\) is small and accordingly small \(\eta\) is sufficient to satisfy the condition 3). These make \(c^T \mathbf{b}(m/g) + \eta\) small, thus forcing the switching function to remain near zero. However, chatter is inevitable because of the discontinuous control. To eliminate the chatter further, a saturation function can replace the sign function. Then, (21) and (22) become
\[
u(k) = -\hat{d}(k) + \left( c^T \mathbf{b} \right)^{-1} \cdot \left[ c^T \mathbf{x}_v(k+1) - c^T \mathbf{A}_g(k) + q\mathbf{s}(k) - \eta \sigma \left( \frac{s(k)}{\phi} \right) \right]
\]
\[
\hat{d}(k) = \hat{d}(k-1) + \left( c^T \mathbf{b} \right)^{-1} \cdot \left[ s(k) - q\mathbf{s}(k-1) + \eta \sigma \left( \frac{s(k-1)}{\phi} \right) \right]
\]
and the following closed-loop dynamics are obtained:
\[
\mathbf{s}(k+1) = q\mathbf{s}(k) - \eta \sigma \left( \frac{s(k)}{\phi} \right) + c^T \mathbf{b} \hat{d}(k)
\]
\[
\hat{d}(k+1) = (1-g) \hat{d}(k) + \hat{d}(k+1) - \hat{d}(k).
\]

Corollary 2 shows that with (28) and (29), \(s(k)\) is still bounded. In fact, it is bounded in the layer with a thickness \(\phi\).
Corollary 2: For (23) in Theorem 3, assume $0 \leq q \leq 1$, $\eta > c^T b(m/g)$, and $|\eta - (\eta/\phi)| < 1$, then, when $|s(k)| > \phi$, $|s(k)|$ decreases until $|s(k)| < \phi$. When $|s(k)| < \phi$, $|s(k)|$ remains smaller than $(c^T b(m/g))/(1 - q + (\eta/\phi))$, which is smaller than $\phi$.

Proof: The proof of the first half of Corollary 2 is trivial. For the second part, change (23) to

$$s(k+1) = \left( q - \frac{\eta}{\phi} \right) s(k) + c^T b \tilde{d}(k)$$

$$= \left[ 1 - \left( 1 - q + \frac{\eta}{\phi} \right) \right] s(k) + c^T b \tilde{d}(k). \quad (32)$$

Then it can be easily proved in the manner similar to Theorem 4 that $|s(k+1)| < (c^T b(m/g))/(1 - q + (\eta/\phi))$ for all $k$ larger than some $k_0$. Also the assumption $\eta > c^T b(m/g)$ implies the following:

$$c^T b \frac{m}{g} < \eta < \eta + \phi(1 - q). \quad (33)$$

Since $\eta$, $\phi$, and $1 - q$ are all positive constants, dividing (33) by $1 - q + (\eta/\phi)$. $(c^T b(m/g))/(1 - q + (\eta/\phi)) < \phi$ is obtained.

Using a saturation function may result in the deterioration of robustness in the conventional DVSC methods. However, the developed DVSC + DDC do not suffer from such a problem. Theorem 5 summarizes the proposed DVSC + DDC with the saturation function.

Theorem 5: For the LTI system (5), choosing the control law and disturbance estimation law, (28), and (29), under the assumptions $|\tilde{d}(k+1) - \tilde{d}(k)| < \eta$, $0 < q < 1$, $|1 - q| < 1$, $|\eta - (\eta/\phi)| < 1$, and $\eta > c^T b(m/g)$, the $s(k)$ dynamics and $\tilde{d}(k)$ dynamics are decoupled and the closed-loop system is always stable regardless of disturbance, and moreover, for a constant disturbance with $m = 0$, the origin of the state vector space is asymptotically stable.

Proof: It is trivial by Theorem 3, Lemma 1, Corollary 2, (30), and (31).

Next, we show the performance of the proposed DVSC + DDC using a numerical example. Consider the system given...
in (5) with

\[
A = \begin{bmatrix} 1.2 & 0.1 \\ 0 & 0.6 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

which is the example in [4]. This example considers the regulation problem with the initial state \( x = [0.3 \ 0.1]^T \). Three controllers are applied to this system with \( c^T = [5 \ 1] \). The example first uses the conventional DVSC [4] with \( qI = 0.25 \) and \( (\eta + \varepsilon)T = 0.25 \). The proposed DVSC + DDC is used second with \( q = 0.75, \eta = 0.043, \) and \( g = 0.5 \). The simulation results are depicted in Figs. 1 and 2. Fig. 1 shows the first two controllers performance in terms of state variables. In Fig. 2, switching functions for both cases are shown, and applied external disturbances is shown with the DDC output. It is shown in Fig. 2 that the effect of external disturbance is compensated for by the DDC. The third controller is also the DVSC + DDC type but uses a saturation function with \( \phi = 0.5 \). Fig. 3 shows this result. The performance of DDC is not presented in the figure when the saturation function is used. As a consequence of the separation principle, it is the same as in the case of using the sign function. Note that the use of saturation function does not significantly deteriorate the robustness of the closed-loop system.

IV. APPLICATION TO A CNC SERVOMECHANISM

A. Modeling the Plant

The performance of the proposed DVSC + DDC controller is experimentally evaluated using a servomechanism, which consists of a 2 kW Yaskawa AC motor (USAMED 20M2) and a factory-designed servopack (CACR-SR20BB1AM). A plant model is obtained by considering the motor and the servopack as a single plant, and an outer-loop controller is added, as shown in Fig. 4. Neglecting the dynamic characteristics of the current controller and the frequency-to-voltage converter in the servopack, which are very fast, the plant can be modeled as a second-order system. Then, the state equations are

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{K_pK_r}{J}(u - x_2) - \frac{f}{J}
\end{align*}
\]

(34)

where \( x_1 \) is the motor position, \( x_2 \) is the motor speed, \( u \) is the speed input, \( f \) is the disturbance or load torque, \( K_p \) is the proportional gain of the servopack speed controller, \( K_r \) is the motor torque constant, and \( J \) is the motor inertia. Defining \( d = -(f/K_pK_r) \), (34) can be rewritten in the matrix form

\[
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K_pK_r}{J} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_pK_r}{J} \end{bmatrix}(u + d).
\]

(35)

Equation (35) can be discretized into the following \( A \) and \( b \) with a sampling time of 1 ms:

\[
A = \begin{bmatrix} 1 & 0.0006 \\ 0 & 0.3074 \end{bmatrix}, \quad b = \begin{bmatrix} 0.0004 \\ 0.6926 \end{bmatrix}.
\]

(36)

The eigenvalues of the matrix \( [I - b(c^Tb)^{-1}c^T]A \) are 0.8127 and 0, which are inside the unit circle in the discrete domain. The parameters for controllers are summarized in Table I. With these controller parameters, the maximum allowed changing rate of the external disturbance \( f \) in (34) is approximately 0.42 Nm in 1 ms. That is to say, \( |f(t + T) - f(t)| < 0.42 \), where \( T = 0.001 \) s.

---

**TABLE I**

<table>
<thead>
<tr>
<th>Controller Parameters</th>
<th>( \eta )</th>
<th>( \phi )</th>
<th>( q )</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DVSC+DDC</td>
<td>1.5854</td>
<td>2.5</td>
<td>1</td>
<td>0.0159</td>
</tr>
</tbody>
</table>

---

**TABLE II**

<table>
<thead>
<tr>
<th>Cutting Conditions</th>
<th>z-axis feed rate</th>
<th>z-axis rotational speed</th>
<th>spindle motor rotational speed</th>
<th>workpiece diameter</th>
<th>drill diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>condition 1</td>
<td>1.67mm/s</td>
<td>10 r/min</td>
<td>1200 r/min</td>
<td>SS41C</td>
<td>70</td>
</tr>
<tr>
<td>condition 2</td>
<td>2.5mm/s</td>
<td>15 r/min</td>
<td>1500 r/min</td>
<td>SS41C</td>
<td>70</td>
</tr>
</tbody>
</table>

---

B. Controller Development

The DVSC + DDC controller is developed following (28) and (29). With the plant (36), the switching function is designed by \( c^T = [210 \ 1] \). Then, the closed-loop dynamics of the error vector can be analyzed using (16)

\[
\begin{bmatrix} I - b(c^Tb)^{-1}c^T \end{bmatrix}A = \begin{bmatrix} 0.8918 \\ -187.2856 \end{bmatrix}.
\]

(37)

The eigenvalues of the matrix \( [I - b(c^Tb)^{-1}c^T]A \) are 0.8127 and 0, which are inside the unit circle in the discrete domain. The parameters for controllers are summarized in Table I. With these controller parameters, the maximum allowed changing rate of the external disturbance \( f \) in (34) is approximately 0.42 Nm in 1 ms. That is to say, \( |f(t + T) - f(t)| < 0.42 \), where \( T = 0.001 \) s.
V. EXPERIMENTAL RESULTS

The proposed controller is implemented on a 486DX2 PC. The schematic of the implementation is shown in Fig. 4. In order to use the required voltage input for the servopack, an input–output (I/O) board is developed. A 24,000-pulse optical encoder is installed for speed measurement. The I/O board is equipped with a 24-bit counter to count encoder pulses for position information, and the M/T method [10] is implemented to obtain a high-accuracy speed signal. The sampling time of this PC-based controller is 1 ms. This controller system is installed on a Tongil TNV-40 CNC milling machine, and cutting experiments were performed. The photograph of the TNV-40 milling machine is shown in Fig. 5. The cutting conditions are summarized in Table II. Position trajectory is simply a ramp function.

In figures, an arrow (↑) is used to indicate the incident of the cutting tool starting to machine the workpiece. Figs. 6 and 7 show that the machining position errors are driven to zero within ±1 or ±2 encoder pulses (equivalent to ±0.417 or ±0.834 μm). Figs. 6 and 7 show that the switching function $s(k)$ remains near zero even at the incident
of machining. Figs. 6 and 7 clearly show the increased load on the motor due to the cutting force at the incident of machining. The increased motor load is also evident at the start. This is due to the acceleration of motor and spindle inertias. Note that the estimated disturbance has no overshoot. An overshoot in the estimated disturbance can have an adverse affect on the system performance. The fast disturbance estimation without an overshoot is possible, because the developed separation principle allows tuning the disturbance estimation dynamics independently from the tracking error dynamics.

VI. CONCLUSION

A new method of achieving the robustness of variable structure control in discrete time is developed. The proposed DVSC + DDC method achieves robust tracking in the presence of an unknown disturbance, which includes external disturbances as well as parameter uncertainties. The proposed method exactly decouples the sliding mode dynamics from the disturbance estimation dynamics, allowing the asymptotic convergence $s(k) \to 0$ and hence $\zeta(k) \to 0$ in both tracking and regulation problems. For this, a bounded changing rate of disturbance
is required, which is generally a more lenient requirement than the conventional requirement of bounded disturbance magnitude. A numerical example as well as experimental results with a 2-kW servomechanism are used to demonstrate the proposed method.

REFERENCES


Yongsoon Eun received the B.S. degree in mathematics from Seoul National University in 1992 and the B.S.E. and M.S.E. degrees in control and instrumentation engineering, also from Seoul National University, Korea, in 1994 and 1997, respectively. Currently, he is a Ph.D. student in the Electrical Engineering and Computer Science Department at the University of Michigan, Ann Arbor.

He served as a Researcher in the Engineering Research Center for Advanced Control and Instrumentation at Seoul National University in 1997. His research interests include variable structure control, vehicle suspension systems, and intelligent transportation systems.

Jung-Ho Kim received the B.S.E. and M.S.E. degrees in control and instrumentation engineering from Seoul National University in 1994 and 1996, respectively. Currently, he is a Ph.D. candidate in the School of Electrical Engineering at Seoul National University, Korea.

His research interests include variable structure control, automotive powertrain systems, intelligent transportation systems, and motion control.

Kwangsoo Kim received the B.S.E. (1996) and M.S.E. (1998) degrees in electrical engineering from Seoul National University, Korea, in 1996 and 1998, respectively. Currently, he is a Ph.D. candidate in the School of Electrical Engineering at Seoul National University.

His research interests include fuzzy systems, motion control, and automotive powertrain systems.


From 1987 to 1993, he was an Assistant Professor in the Mechanical and Aerospace Engineering Department at Princeton University, Princeton, NJ. In 1993, he joined the Department of Control and Instrumentation Engineering at Seoul National University, Korea. Currently, he is an Associate Professor in the School of Electrical Engineering. His research interests are in variable structure control, mechatronics, ITS, and MEMS.

He was an Associate Editor for the *IEEE/ASME JOURNAL OF MICROELECTROMECHANICAL SYSTEMS* from 1992 to 1997 and the *IOP Journal of Micromechanics and Microengineering* from 1992 to 1997. He also served as an Acting Associate Editor for the *ASME Journal of Dynamic Systems, Measurement, and Control* in 1993.